AP Calculus BC

Particle Motion

1) $\int_0^{15} |v(t)|\,dt$

$= \frac{1}{2}[v(0)+v(2)]\cdot 2 + \frac{1}{2}[v(2)+v(5)]\cdot 3 + \frac{1}{2}[v(5)+|v(6)|]\cdot 1 + \frac{1}{2}[|v(6)|+v(8)]\cdot 2$
$+ \frac{1}{2}[v(8)+|v(12)|]\cdot 4 + \frac{1}{2}[|v(12)|+v(15)]\cdot 3$

$= [10] + \frac{3}{2}[15] + \frac{1}{2}[14] + [18] + 2[22] + \frac{3}{2}[19]$

represents the total distance traveled by the object from $t=0$ to $t=15$.

2) a) $s(4)-s(0) = 0.2-40.0$
$= -39.8$

b) Twice $s(t)$ changes from dec to inc on $[0, 1.5]$ & from inc to dec on $[1.5, 2.7]$ which means that $v(t)$ changes signs.

c) $\frac{s(0.5)-s(0)}{0.5-0} = \frac{35-40}{0.5}$
$= 10$

d) $v(0.5) \approx \frac{s(1)-s(0)}{1-0} = -9.8$

$v(2.7) \approx \frac{s(3)-s(2)}{3-2} = -10$

$v(3.5) \approx \frac{s(3.6)-s(3.0)}{3.6-3.0} = \frac{16.0-38.2}{0.6}$

3) a) $t=1$ & $t=4.1$, $v(t)=0$
b) $(0,1)$ & $(4.1,6)$ $v(t)>0$
c) max speed @ $t=3$
max vel @ $t=0$ & $(5,6]$
d) speed inc on $(1,2)$ & $(2,3)$ & $(4.1,5)$
$v(t)$ & $a(t)$ have the same sign
speed dec on $(0,1)$, $(3,4.1)$
$v(t)$ & $a(t)$ have opp signs
e) $a(4.8) = 5$ ft/sec^2
$v(t)$ is increasing
f) max acc on $(3,5)$
g) $\int_0^2 v(t)\,dt = 0$

4) a) $(0,1)$ & $(5,7)$ $v>0$
b) $(1,5)$ $v<0$
c) $t=1$ & $t=5$, $v(t)$ Δs signs
d) $a>0$ on $(3,6)$
$a<0$ on $(0,2)$
e) $\int_0^9 v(t)\,dt = -2 - \frac{1}{2}(2)(2) + \frac{1}{2}(2)(1)$
f) $s(4) = 4 + \int_0^4 v(t)\,dt$
$= 4 - \left[2 - \frac{1}{2}[2+1]\cdot 1\right]$
g) $\int_0^9 |v(t)|\,dt = 2+2+2+\frac{1}{2}(2)(2)+\frac{1}{2}(2)(1)$

5) a) $v'(16) \approx \frac{v(20)-v(12)}{20-12}$

$= \frac{240-200}{20-12}$

b) $\int_0^{40} |v(t)|\,dt$ represents the total distance, in meters, that Johanna jogs from $t=0$ to $t=40$ min

$\int_0^{40} |v(t)|\,dt = v(12)\cdot 12 + v(20)\cdot 8 + |v(24)|\cdot 4 + v(40)\cdot 16$

$= (200)(12) + (240)(8) + (220)(4) + (150)(16)$

c) $B'(t) = 3t^2 - 12t$

$B'(5) = 3(25) - 12(5)$

d) $\frac{1}{10}\int_0^{10} (t^3 - 6t^2 + 300)\,dt$

$\frac{1}{10}\left[\frac{1}{4}t^4 - 2t^3 + 300t\right]\Big|_0^{10}$

$\frac{1}{10}\left[\frac{1}{4}(10)^4 - 2(10)^3 + 300(10)\right]$